The resonance frequency of a membrane absorber

By M. Feuerbacher, April 2005

Bass traps for listening rooms in the form of membrane absorbers are highly popular due to their simple construction and their broad bandwidth. Numerous equations for the calculation of the resonance frequencies of such absorbers are found on the internet. However, there are many different equations around, some of which are in obvious mutual contradiction. It is not clear which equations can be used and which ones are simply wrong. Furthermore, the equations are frequently given in numerical form, but it is not stated in which units the parameters have to be entered. I could not find a source for a derivation of the resonance frequency. Therefore I decided to calculate the resonance frequency of a panel absorber by myself and present the result in this note.

A panel absorber consists of a gas filled cavity, a rigid resonator body, closed by a membrane. Oscillations of the membrane are excited by incoming sound waves. The motion of the membrane leads to compression of the gas in the absorber. The energy of the incoming wave is dissipated as heat in the enclosed volume.

The absorber can be treated as a spring-mass system. It has a resonant frequency at which it works most effectively. The absorber can therefore be tuned by the geometry of the system and can be adapted to the desired frequency range. An exact treatment of the system is very difficult. In the following, an approximation to the problem is given, which will however lead to a practically usable result.

Consider an absorber of (internal) thickness t and a membrane of mass m and area A. The volume of the included air is then given by V = At. As an approximation we assume that the membrane oscillates as a whole, that is, it moves without deformation into the rigid resonator body and compresses the enclosed volume of air (Fig. 1).

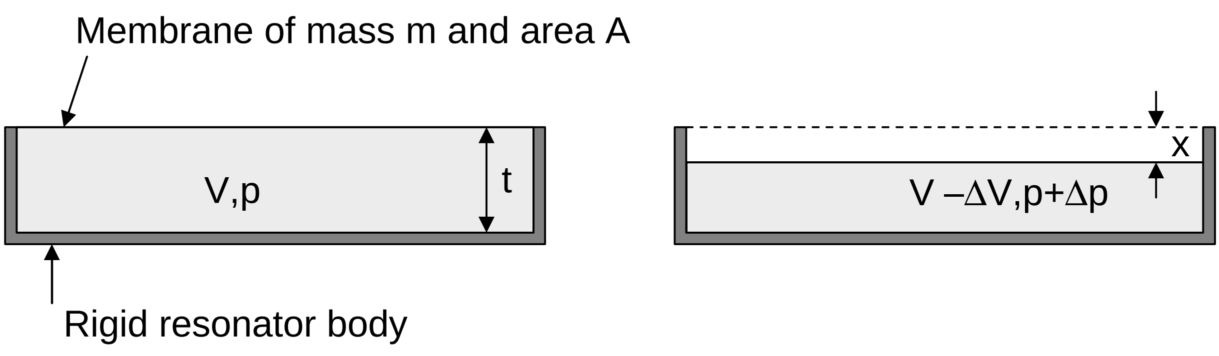

Fig. 1: Schematic of the membrane resonator

As the membrane moves by a small distance into the resonator body, the included volume V is reduced by $\Delta V = Ax$. This leads to a pressure increase Δp in the gas and accordingly the force $F = \Delta p \cdot a$ acts on the membrane. Let us set up an equation of motion using Newtons first law $F = m\, d^2x/dt^2$, where m is the mass of the membrane. The mass of the membrane can generally be written as $m = \rho_A \cdot A$, where ρ_A is the area density (mass per area) of the membrane material. Putting all this together we obtain

$$\frac{d^2x}{dt^2} = \frac{\Delta pA}{\rho_A A} = \frac{\Delta p}{\rho_A} \qquad (1)$$

The gas in the volume is compressed adiabatically (i.e. so fast that the heat cannot be transported away). Therefore the standard relation for ideal gases

$$\frac{\Delta p}{p} = \kappa \frac{\Delta V}{V}, \qquad (2)$$

where p and V are the initial pressure and volume, respectively, and γ is the ratio of specific heats at constant pressure and volume of the gas. Inserting the volumes we obtain

$$\frac{\Delta p}{p} = \kappa \frac{A \cdot x}{A \cdot t}, \qquad (3)$$

which leads to an expression for the pressure change Δp. Combining (1) and (3) we arrive at the relation

$$\frac{d^2x}{dt^2} = -\frac{\kappa p}{\rho_A t} x. \qquad (4)$$

This is the equation of an harmonic oscillator (the restoring force is proportional to the displacement). The frequency f of the oscillator, corresponding to the eigenfrequency of the absorber, is then given by

$$f = \frac{1}{2\pi} \sqrt{\frac{\kappa p}{\rho_A t}}. \qquad (5)$$

Finally, we can express the velocity of sound in a gas as a function of the density of the gas ρ, the pressure and the ratio of the heat capacities. $c = \sqrt{\kappa p / \rho}$. This leads us to the final result

$$f = \frac{c}{2\pi} \sqrt{\frac{\rho}{\rho_A t}}. \qquad (6)$$

with: c: sound velocity in the gas
ρ: mass density of the gas
ρ_A: area density (mass per area) of the membrane
t: inner thickness of the resonator

This general equation can be transformed into a numerical form for practical use. Commonly, the cavity is filled with air. The density of air at room temperature is about 1.29 kg/ m^3 and the speed of sound is 344 m/s. Equation (6) can then be written as

$$f = 62 \frac{1}{\sqrt{\rho_A t}}, \qquad (7)$$

where we have calculated the constant as $\dfrac{344 \frac{m}{s} \cdot \sqrt{1.29 \frac{kg}{m^3}}}{2\pi}$. So note that this constant has the unit $s^{-1}\sqrt{m/s}$. The complete expression thus yields the correct unit of a frequency, s^{-1} or Hz. Note furthermore, that with the transition from (6) to (7), we have fixed the units. The area density has to be entered in kg/m^2 and the thickness of the absorber in m. Equation (7) is one of the expressions frequently found in the internet. Now you know where it comes from.

Tuning of the resonator can be done by variation of the geometry of the resonator (a thicker body results in a lower resonant frequency) and by the mass density of the membrane (a heavier membrane will lead to a lower frequency). Note however, that this does *not* mean that you should use a *thicker* membrane. With a thick membrane the approximation made for the derivation will be less precise (if not fully violated) and the resonance frequency will be different (see below) and coupling to the incoming sound wave will be worse. You can, however, use a material with a high mass density, e.g. a steel plate. There are also some less obvious ways to tune the absorber. Filling the cavity with a heavier gas will lead to a lower frequency. This, however, is a tuning means not very useful in practice.

Notes on the approximation made:
For the derivation presented, it is assumed that the membrane moves as a whole without any deformation. This, of course is a crude approximation of what happens in reality. Since the real membrane is fixed to the rigid resonator body, it will deform if subjected to the incoming sound wave and oscillates like the string of a guitar or a violin (Fig. 2).

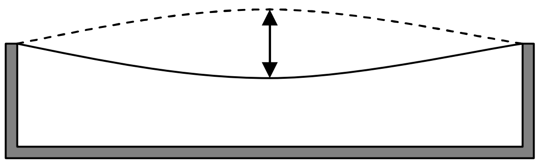

Fig. 2: More realistic vibration mode of the membrane

The resonant frequency will therefore be determined not only by the compression of the enclosed gas but also by the elastic moduli and the geometry (thickness and size) of the membrane. Due to the weak coupling of the incoming sound wave and the membrane, the latter will vibrate in its lowest eigenmode, i.e. the lateral dimension of the membrane will be equal to half a wavelength. For this lowest eigenmode, the above approximation of a rigid membrane will be better than for the higher eigenmodes.For a bass trap, the frequency range one wants to absorb are of the order of some ten to some hundred Hz. This corresponds to rather large wavelengths of the membrane. Therefore its lateral dimensions should not be too small. A *large membrane* will facilitate coupling to the incoming sound wave. These arguments correspond to comments made that the edge lengths of the absorber should exceed 0.5 m.
The frequently quoted preferable aspect ratio of 1:1.5 to 1:1.7 can *not* be reproduced from the above treatment. According to the calculation, an aspect ratio of say 1:1 should be equally effective.

Damping:
The volume in the cavity can be filled with damping material, e.g. rigid fiberglass. Damping will have the following results: First, the bandwidth of the resonator will be wider (a general feature of damping of an oscillator). Second, the dissipation of energy in the cavity will be more effective.